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Subspace Method for the Estimation of Large-Scale Structured Real Stability Radius

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We consider the autonomous dynamical system $x' = Ax$, with $A \in \mathbb{R}^{n\times n}$. This linear dynamical system is *asymptotically stable* if all of the eigenvalues of A lie in the open left-half of the complex plane. In this case, the matrix A is said to be *Hurwitz stable* or shortly a *stable* matrix. In practice, the stability of a system can be violated because of perturbations such as modeling errors. In such cases, one deals with the robust stability of the system rather than its stability. The system above is said to be *robustly stable* if the system, as well as all of its perturbations from a certain perturbation class, are stable. To measure the robustness of the system subject to perturbations, a quantity of interest is the *stability radius* or in other words the *distance to instability*. In this talk, we focus on the estimation of the structured real stability radius for large-scale systems. We propose a subspace framework to estimate the structured real stability radius and prove that our new method converges at a quadratic rate in theory. Our method benefits from a one-sided interpolatory model order reduction technique, in the sense that the left and the right subspaces are the same. The quadratic convergence of the method is due to the certain Hermite interpolation properties between the full and reduced problems. The proposed framework estimate the structured real stability radius for large-scale systems efficiently. The efficiency of the method is demonstrated on several numerical experiments.

Key words. real stability radius, structured, large-scale, projection, singular values, Hermite interpolation, model order reduction, greedy search.

Assessment of locally resonant structures using adaptive parametric model order reduction

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Locally resonant materials, sometimes termed “metamaterials”, are a tool to reduce unwanted structural vibration of mechanical systems in a specific frequency range. Such materials consist of a periodic array of vibrating substruc-

tures attached to the host structure whose vibration response is to be modified. The frequency range in which the vibration of the host structure is reduced can be modified by tuning the geometric or material parameters of the substructures. This concept is similar to so called tuned vibration absorbers (TVA), which are used to attenuate vibration of buildings or bridges.

In order to gain an optimal reduction of the host structure's vibration response in a defined frequency range, the parameters of the substructures need to be tuned accordingly. In this experiment, we examine a flexible plate as host structure and attach an array of discrete mass-spring-damper systems as resonant substructures. The additional mass introduced by the substructures is fixed to a fraction of the plate's mass and we optimize the subsystems' stiffness and damping coefficients to reduce the vibration of the host structure in a defined frequency range.

Model order reduction methods can be employed to reduce the computational effort required by the repeated evaluation of the numerical model of the locally resonant material during an optimization. Parametric model order reduction (PMOR) techniques compute surrogates approximating the original system while retaining the dependency on certain parameters. We employ a moment matching method to obtain a surrogate model depending on the material parameters of the resonant substructures. Using this surrogate, these parameters can efficiently be optimized to minimize the vibration of the host structure in a specified frequency region. Reasonable locations for the expansion points required for moment matching are computed automatically using a greedy method.

Structure-preserving data-driven modeling for mechanical systems

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Data-driven and reduced-order modeling are essential tools in the construction of high-fidelity compact dynamical models that approximate real-world physical phenomena. The setup is as follows: an explicit model (e.g., a linear time-invariant system) described by a state-space formulation with access to internal variables is not available, but instead, access to input/output data is granted. In many applications, it is essential to inherit physical meaning and structure in the learned reduced-order model that allows the use of appropriate computational tools and physical re-interpretation. Out of the many methods for approximating dynamical systems that are based on data, we mention here the vector fitting (VF) algorithm in [1], and also the AAA (Adaptive Antoulas-

Anderson) algorithm in [2]. In a nutshell, these are based on iteratively fitting a rational approximant to the data, by means of solving linearized least-squares problems, and by using the barycentric representation of the rational transfer function.

We develop data-driven structure-preserving modeling frameworks for mechanical systems described by second-order dynamics. Here, data represent transfer function samples of the underlying mechanical system. We develop a structure-preserving formulation of the data-driven VF and AAA algorithms for the cases of modally damped [3] or proportionally (Rayleigh) damped [4] mechanical systems. We propose various structured extensions for the classical barycentric formula of the system's transfer functions. Integrating these new reformulations within the classical VF and AAA algorithms leads to new algorithms that allow the computation of reduced-order mechanical systems from data in a least-squares fashion. Thus, the learned models are guaranteed to have the desired structure. We test the new algorithms on various benchmark models and data sets.

References

[1] Gustavsen, B. and Semlyen, A., *Rational approximation of frequency domain responses by vector fitting*. IEEE Trans. Power Del., 14(3), 1052–1061, 1999.

[2] Nakatsukasa, Y., Sete, O., and Trefethen, L.N., *The AAA algorithm for rational approximation*. SIAM J. Sci. Comput., 40(3), A1494–A1522, 2018.

[3] Werner, S.W.R., Gosea, I. V., Gugercin, S., *Structured vector fitting framework for mechanical systems*. e-print 2110.09220, arXiv. URL http://arxiv.org/abs/2110.09220. Math.NA, 2021.

[4] Benner, P., Goyal, P., and Pontes Duff, I., *Data-driven identification of Rayleigh-damped second-order systems*. e-print 1910.00838, arXiv. URL http://arxiv.org/abs/1910.00838.

Learning for Low-Dimensional Nonlinear Dynamical Systems Using Operator Inference

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Dynamical modeling of a process is essential to study its dynamical behavior and perform engineering studies such as control and optimization. With the ease of accessibility of data, learning models directly from the data have recently drawn much attention. It is also desirable to construct compact low-dimensional models describing complex nonlinear dynamics, leading to simulations and engineering studies on modest computer hardware. This talk presents a data-driven operator inference (OpInf) approach for the nonlinear system to learn low-dimensional models. Here, it is assumed that the structure of the non-linearity is known only at an abstract level. In light of this, we discuss how to tailor the OpInf approach to get interpretable physical models. Moreover, we

discuss the usage of deep learning to improve the performance of the approach further, particularly in the spirit of nonlinear projection using autoencoders. We illustrate the methodologies to learn low-dimensional dynamical models by means of often encountered engineering problems.

Physics informed Tensor Network and Deep Neural Network Models for Approximating Eigenmodes

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We study eigenmodes of a class of elliptic reaction-diffusion operators. The prototype model problem will be a family of Schrdinger Hamiltonians, but we will also consider problems with highly oscillating coefficients. The problem which we will consider is posed in the finite domain and we compute localized bounded states at the lower end of the spectrum. For tackling higher dimensional problems, we consider a class of physics-informed deep dense networks. Also, we will consider broad shallow networks where we compress the network matrix using the tensor train decomposition. We will focus on the interpretability of the proposed approaches. The performance of the neural network surrogate reduced model is controlled by an error estimator and we will show how to use standard residual error estimators in this context. Finally, we present a host of experiments to measure the accuracy and performance of the proposed algorithm.

A data-driven formulation for balanced truncation of linear and bilinear dynamical systems

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Balanced truncation (BT) [2, 3] is one of the gold standards in model reduction of dynamical systems. BT has been traditionally a projection-based

approach, i.e., it requires access to the full-order state-space matrices to construct the reduced model. In the first part of the talk, following our recent work [1] we present a novel data-driven reformulation of BT for linear dynamical systems. This new formulation of BT only assumes access to system response data that has been either measured or computed, without reference to any prescribed realization of the original model. Data are represented by sampled values of the transfer function or the impulse response corresponding to the original model. In the second part of the talk, building on [1], we extend the analysis and methodology to bilinear control systems and describe a non-intrusive data-driven formulation of BT for bilinear control systems. Throughout the talk, we illustrate the theory via various numerical examples.

References

[1] Gosea, I. V., Gugercin, S., Beattie., C. A. *Data-Driven Balancing of Linear Dynamical Systems. SIAM Journal on Scientific Computing*, 44(1), A554-A582 (2022).

[2] Moore, B. *Principal component analysis in linear systems: Controllability, observability, and model reduction.* IEEE Transactions on Automatic Control 26(1), 17-32 (1981).

[3] Mullis, C., Roberts, R. *Synthesis of minimum roundoff noise fixed point digital filters.* IEEE Transactions on Circuits and Systems, 23(9), 551-562(1976).

Fiedler-like structure preserving strong linearizations for structured matrix polynomials

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Matrix polynomials arise in many application and linearization is a widely used method for solving polynomial eigenproblems in which a matrix polynomial is transformed to a matrix pencil of larger size. Structure preserving linearizations of structured (symmetric, skew-symmetric, palindromic, even, odd) matrix polynomials have been proposed in the literature so as to preserve the spectral symmetry in the eigenvalues. We present a new family of structured strong linearizations of a structured matrix polynomial for various structure matrix polynomials. Mainly, we focus on a new family of palindromic strong linearizations of a palindromic matrix polynomial of odd degree. A salient feature of the new family is that it allows the construction of banded palindromic linearizations of block-bandwidth $k+1$ for any $k = 0 : m-2$, where m is the degree of

the palindromic matrix polynomial. Low bandwidth palindromic pencils may be useful for numerical computations. Our construction of the new family is based on Fiedler companion matrices associated with matrix polynomials and the construction is operation-free. Moreover, the new family of palindromic pencils allows operation-free recovery of eigenvectors and minimal bases, and an easy recovery of minimal indices of matrix polynomials from those of the palindromic linearizations. We also present an operation-free algorithm for construction of palindromic pencils belonging to the new family.

Perturbation bounds for a stable gyroscopic systems

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In this talk we will consider a linear gyroscopic mechanical systems of the form

$$M\ddot{x}(t) + G\dot{x}(t) + Kx(t) = 0, \tag{1}$$

where the mass matrix $M \in \mathbb{R}^{n\times n}$ and the stiffness matrix $K \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices while the gyroscopic matrix $G \in \mathbb{R}^{n\times n}$ is a skew-symmetric i.e. $G^T = -G$ and $x = x(t)$ is a time-dependent displacement vector.

More precisely, we will consider the perturbation theory for the gyroscopic systems of the form (1). We will present new relative perturbation bounds for the eigenvalues as well as the $\sin\Theta$ type bound for the perturbation of the corresponding eigenspaces. Obtained results will be illustrated in the numerical experiments.

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References

[1] I. Kuzmanović Ivičić, S. Miodragović. Perturbation bounds for a stable gyroscopic systems, submitted

[2] N. Truhar, S. Miodragović. Relative perturbation theory for definite matrix pairs and hyperbolic eigenvalue problem. *Applied Numerical Mathematics, 98:106121, 2015.*

The tan Θ theorem for definite matrix pairs

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In this talk we will consider the perturbation of a Hermitian matrix pair (H, M), where H and M are non-singular and positive definite Hermitian matrices, respectively. A novel upper bound on a tangent of the angles between the invariant subspaces of perturbed and unperturbed pairs will be derived under a perturbation to the off-diagonal blocks of H. The rotation of the invariant subspaces under a perturbation will be measured in the matrix-dependent scalar product.

We will show that the $\tan\Theta$ bound for the standard eigenvalue problem is a special case of our new bound and that obtained bound can be much sharper than the existing $\sin\Theta$ bounds.

Structured $\mathcal{L}_2$-Optimal Parametric Model Order Reduction

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Motivated by $\mathcal{H}_2$-optimal model order reduction (MOR) for non-parametric linear time-invariant (LTI) systems and $\mathcal{H}_2 \otimes \mathcal{L}_2$-optimal parametric MOR for parametric LTI systems, we investigate $\mathcal{L}_2$-optimal parametric MOR for parametric stationary problems arising from, e.g., discretizations of parametric stationary partial differential equations.

We first develop gradients of the squared $\mathcal{L}_2$ error with respect to the reduced system operators, which then leads to a gradient-based optimization method for MOR of parametric stationary problems. We also illustrate that the optimization algorithm can be performed purely in a data-driven manner using only the samples of the quantities of interest without access to full-order operators.

Next, we generalize the theory to complex-valued and unbounded parameter sets, leading to a unifying framework for $\mathcal{L}_2$-based MOR of stationary problems and dynamical systems. In particular, we show that this unifying framework recovers the well-known optimality conditions for $\mathcal{H}_2$ and $\mathcal{H}_2 \otimes \mathcal{L}_2$-optimal MOR. Furthermore, we develop interpolatory conditions for optimal MOR of a class of parametric stationary problems. Finally, we discuss MOR methods based on (Petrov-)Galerkin projection and whether $\mathcal{L}_2$-optimal reduced-order models are necessarily of such type.

We illustrate the theory via various numerical examples and compare our framework to standard projection-based approaches.

Optimal control of parabolic equations – a spectral calculus based approach

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We consider an optimal control problem for a linear parabolic equation governed. We represent the solution operator for this problem as a function of a self-adjoint operator on an abstract Hilbert space. The task consists in identifying a control (entering the system through the initial condition) that minimizes the distance of the trajectory of the system from a given constraint while steering the final state at time $T > 0$ close to the given target. We present the closed form solution for several types of trajectory constraints and then develop a numerical scheme based on the `rkfit` algorithm for approximating the trajectory of the system using spectral calculus and a rational function surrogate.

This talk is based on joint work with Luka Grubišić, Martin Lazar, and Martin Tautenhahn.

Finite time horizon mixed control of vibrational systems

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In this talk we consider a vibrational system control problem over a finite time horizon. The performance measure of the system is taken to be p-mixed H_2 norm which generalizes the standard H_2 norm. We present an algorithm for efficient calculation of this norm in the case when the system is parameter dependent and the number of inputs and outputs of the system is significantly smaller than the order of the system. Our approach is based on a novel procedure which is not based on solving Lyapunov equations and which takes into account the structure of the system. We use a characterization of the H_2 norm given in terms of integrals which we solve using adaptive quadrature rules. This enables us to use recycling strategies as well as parallelization. The efficiency of the new algorithm allows us to analyse the influence of various system parameters and different finite time horizons on the value of the p-mixed H_2 norm. We illustrate

our approach by numerical examples concerning an n-mass oscillator with one damper.

Full state approximation via Gramian-based Galerkin projection

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In this talk, the problem of full state approximation by model reduction is studied for deterministic and stochastic linear systems and bilinear systems. Our proposed approach relies on identifying the dominant subspaces based on the reachability Gramian. Once the desired subspace is computed, the reduced-order model is then obtained by a Galerkin projection. We prove that this approach either preserves asymptotic stability or leads to reduced models whose minimal realization is asymptotically stable. Moreover, this stability preservation guarantees the existence of the reduced system reachability Gramian which is the basis for the full state error bounds that we derive. This error bound depends on the neglected eigenvalues of the reachability Gramian and hence shows that these values are a good indicator for the expected error in the dimension reduction procedure. We conclude the paper by numerical experiments using a benchmark problem. Finally, we compare this approach with balanced truncation and show that it performs well in reproducing the full state of the system.

Semi-active Damping Optimization of Vibrational Systems using the Reduced Basis Method

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In this work, we consider the stabilization of vibrational systems with semi-active damping, as they occur in civil engineering applications, such as buildings, bridges, or dams. To this aim, dampers are installed to avoid strong movements caused by external forces with frequencies close to the eigenfrequencies of the structure. Our main goal is to optimize the damping matrix and the included

damping gains in order to minimize the influence of the input on the output. As minimization criterion, we evaluate the energy response, that is, the $\mathcal{H}_2$-norm of the corresponding transfer function of the system. The computation of the energy response involves the solution of a Lyapunov equation. Therefore, the minimization process leads to high computational costs when the system is of large dimension.

In order to speed up this process, we develop different techniques that use the reduced basis method to generate a basis that spans an approximation of the solution space of the Lyapunov equations for all parameters of interest. Using this basis, we formulate a reduced optimization problem whose function values can be computed quickly, resulting in a significant speed-up in the minimization process. To evaluate the quality of the approximation, we use an error estimator that evaluates the error in the energy response. Based on this, we propose two different methods. In the first method, we initially determine a basis and then initiate the optimization process; in the second method, the bases are enriched during the optimization process. Finally, we apply the two methods to examples and illustrate their merits.

Auto-adaptive Moment Matching-type PMOR via low-rank (tensor) approximation

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Moment Matching via rational Krylov subspaces is usually computed by basis creation from Arnoldi-type Algorithms. The key ingredient is the selection of good shifts in the Krylov subspace. The essential idea is to use significant sampling frequencies of the transfer function, to capture its important features.

It is a well known fact that the same basis could be computed from the solution of a Sylvester matrix equation with large an sparse matrices on the left and possibly dense and small matrices on the right.

Different methods for the shift optimization exist. In the iterative rational Krylov Algorithm (IRKA) these are, for a fixed problem size, successively adapted to optimize the $\mathcal{H}_2$-error. Other methods use a-posteriori error estimators in a greedy optimization procedure.

Our contribution aims to use drastic oversampling of the transfer function and, consequently, Sylvester equations with large and sparse coefficients on both sides. If the transfer function is well-natured enough, the solution has a strong singular value decay. This can be used to actually compute low-rank solutions and, thus, potentially much smaller projection spaces.

For parametric systems the Sylvester equations become multi-term Sylvester equations and require low-rank tensor methods

Singular quadratic eigenvalue problems: Linearization and weak condition numbers

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In this talk we will study the numerical computation of singular quadratic eigenvalue problem associated with matrix polynomial $Q(\lambda) = \lambda^2 M + \lambda C + K$ such that $\det(Q(\lambda)) \equiv 0$. Since small matrix perturbation of singular problems can move eigenvalues anywhere in the complex plain these problems are known to be ill–conditioned. However, it was recognized by Wilkinson [3], and later proven by De Terán, Dopico and Moro [1] that perturbation directions causing arbitrary large eigenvalue changes are rare.

More recently, in order to quantify the eigenvalue sensitivity, Lotz and Noferini [2] considered perturbation directions which are uniformly distributed on the sphere. They introduced the so called δ–weak condition number that bounds the eigenvalue sensitivity with probability $1 - \delta$, $0 \leq \delta < 1$, together with an upper bound.

Since linearization is logical first step for the solution of polynomial eigenvalue problems, we supplement the theory of δ–weak condition number with its lower bound in order to prove that one can always choose the linearization for this problem so that this quantity is not affected. In addition, we develop theoretical criterion for classification of eigenvalues of a perturbed matrix polynomial leading to a new procedure for the solution of singular quadratic eigenvalue problem. We prove that well–conditioned finite eigenvalues can be detected with high probability. Finally, we will present numerical experiments to demonstrate the efficiency of proposed method.

References

[1] Fernando De Teráan, Froilán M. Dopico, and Julio Moro. First order spectral perturbation theory of square singular matrix pencils. *Linear Algebra Appl.*, 429(2-3):548576, 2008.

[2] Martin Lotz and Vanni Noferini. Wilkinsons bus: weak condition numbers, with an application to singular polynomial eigenproblems. *Found. Comput. Math.*, 20(6):1439 1473, 2020.

[3] J. H. Wilkinson. Kroneckers canonical form and the QZ algorithm. *Linear Algebra Appl.*, 28:285303, 1979.

Numerically Efficient Agents-to-Group H_∞ Analysis

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We consider a numerically efficient approach for computing the maximal/minimal impact a subset of agents has on the cooperative system. For instance, if one is able to disturb/bolster several agents so as to maximally disturb/bolster the entire team. Important questions are which agents to choose and what kind of inputs to apply? We quantify the agents-to-team impacts in terms of H_∞ norm whereas output synchronization is taken as the underlying cooperative control scheme.

Sufficient conditions on agents' parameters, synchronization gains and topology are provided such that the associated H_∞ norm attains its maximum for constant agents' disturbances. Linear second-order agent dynamics and weighted undirected topologies are considered. Our analyses also provide directions towards improving graph design and tuning/selecting cooperative control mechanisms.

Lastly, we present several numerical examples that illustrate proposed approach.

Optimal positioning of dampers in mechanical systems

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We consider the damping optimization problem for systems defined by the vector differential equation $M\ddot{x} + C\dot{x} + Kx = 0$, where $M, C, K \in \mathbb{R}^{n\times n}$ are mass, damping and stiffness matrices, respectively.

We present a novel approach for simultaneous optimization of positions and damping viscosities. The positions' optimization is based on two new results for the cases without and with internal damping, respectively.

Although the problem of the best placement of dampers is very demanding, we will present a new proof for the well-known result on the best position for one damper which damped just one undamped frequency.

On the other hand, if we try to damp more than one frequency with two or more dampers, then we will present a new heuristic for defining a feasible set of possible optimal positions. It is based on the local minimizers for the one-dimensional damping acting on only one dangerous (or dominant) frequency. For this case, we also present a quality analysis for the trace function $f(v) = \text{trace}(X(v))$, which allows us to approximate the trace function as a rational function.

Presented results are joint work with:

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Fast optimization of viscosities for frequency-weighted damping of second-order systems

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We consider frequency-weighted damping optimization for vibrating systems described by a second-order differential equation. The goal is to determine viscosity values such that eigenvalues are kept away from certain undesirable areas on the imaginary axis. To this end, we present two complementary techniques. First, we propose new frameworks using nonsmooth constrained optimization problems, whose solutions both damp undesirable frequency bands and maintain stability of the system. These frameworks also allow us to weight which frequency bands are the most important to damp. Second, we also propose a fast new eigensolver for the structured quadratic eigenvalue problems that appear in such vibrating systems. In order to be efficient, our new eigensolver exploits special properties of diagonal-plus-rank-one complex symmetric matrices, which we leverage by showing how each quadratic eigenvalue problem can be transformed into a short sequence of such linear eigenvalue problems. The result is an eigensolver that is substantially faster than existing techniques. By combining this new solver with our new optimization frameworks, we obtain our overall algorithm for fast computation of optimal viscosities. The efficiency and performance of our new methods are verified and illustrated on several numerical examples.

Structure-Preserving Balanced Truncation Model Reduction for Symmetric Second-Order Systems

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We introduce a model reduction approach for linear time-invariant second-order systems based on positive real balanced truncation. Our method guarantees to preserve passivity of the reduced-order model as well as the positive definiteness of the mass and stiffness matrices and admits an a priori gap metric error bound. Our construction of the second-order reduced model is based on the consideration of an internal symmetry structure and the invariant zeros of the system and their sign-characteristics for which we derive a normal form. The results are available in [1].

References

[1] I. Dorschky, T. Reis, and M. Voigt. Balanced truncation model reduction for symmetric second order systems – a passivity-based approach. *SIAM J. Matrix Anal. Appl.*, 42(4):1602–1635, 2021.

A New Tangential Interpolation Framework for Model Reduction of Bilinear Systems

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The modeling of natural processes as population growth, mechanical structures and fluid dynamics, or stochastic modeling often results in bilinear time-invariant dynamical systems:

$$E\dot{x}(t) = Ax(t) + \sum_{j=1}^{m} N_j x(t) u_j(t) + Bu(t), \quad y(t) = Cx(t), \tag{2}$$

with $E, A, N_j \in \mathbb{R}^{n\times n}$, for $j = 1, \ldots, m$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$. Model reduction is an essential tool to reduce the amount of computational resources such as time and memory needed for the use of such systems (2) in computer-aided design cycles by constructing cheap-to-evaluate surrogates with a significantly smaller number of internal states, $r \ll n$. Using Volterra series expansions [2], an equivalent description of (2) is given in the frequency domain in terms of multivariate, rational transfer functions:

$$G_k \colon \mathbb{C}^k \to \mathbb{C}^{m^k}, \tag{3}$$

with $k \in \mathbb{N}$. In the single-input/single-output case ($m = p = 1$), the transfer functions (3) are scalar for all k and interpolation of these is an efficient and successful model reduction approach. However, in the multi-input/multi-output case ($m, p > 1$), the transfer functions (3) are matrix-valued and increase exponentially in size with the level k. Classical (matrix) interpolation leads consequently to potentially very large reduced-order models; see [1].

For linear dynamical systems, i.e., $N_j = 0$ in (2), tangential interpolation provides an effective solution to the dimension growth in the reduced-order model by restricting matrix interpolation problem into chosen evaluation directions (tangents). In this talk, we present a new framework for tangential interpolation of bilinear systems (2). This enables us to efficiently construct reduced-order models via interpolation by resolving the problem of the exponentially increasing number of interpolation conditions in (3).

References

[1] P. Benner, S. Gugercin, and S. W. R. Werner. Structure-preserving interpolation of bilinear control systems. *Adv. Comput. Math.*, 47(3):43, 2021.

[2] W. J. Rugh. *Nonlinear System Theory: The Volterra/Wiener Approach.* The Johns Hopkins University Press, Baltimore, 1981.