Polynomial eigenvalue problems and Hermitianity

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In this talk we consider the problem of perturbations of the invariant pair $(X, S) \in \mathbb{C}^{n\times k} \times \mathbb{C}^{k\times k}$ of a polynomial eigenvalue problem

$$\mathbf{P}(X, S) \doteq A_n X S^n + A_{n-1} X S^{n-1} + \ldots + A_1 X S + A_0 X = 0$$

with general coefficient matrices $A_i \in \mathbb{C}^{n\times n}$, $i = 0, 1, \ldots n$. The corresponding perturbed problem is

$$\widetilde{\mathbf{P}}(\widetilde{X}, \widetilde{S}) \doteq \widetilde{A}_n \widetilde{X} \widetilde{S}^n + \widetilde{A}_{n-1} \widetilde{X} \widetilde{S}^{n-1} + \ldots + \widetilde{A}_1 \widetilde{X} \widetilde{S} + \widetilde{A}_0 \widetilde{X} = 0$$

where $\widetilde{A}_i \in \mathbb{C}^{n\times n}$, for $i = 1, \ldots n$ are general as well.

We present a novel upper and lower bound for the $\|\sin \Theta_H(X, \widetilde{X})\|_F$ and $\|\cos \Theta_H(X, \widetilde{X})\|_F$, where $\Theta_H(X, \widetilde{X})$ is a matrix of canonical angles between column subspaces spanned with X and $\widetilde{X}$, given in appropriate H-definite product, respectively.

Also we will suggest novel algorithm called *optimal quotient iteration* for calculation of eigenpairs of corresponding quadratic eigenvalue problem

$$P_2(\lambda)x \doteq (\lambda^2 A_2 + \lambda A_1 + A_0)x = 0\,,$$

where $A_0, A_1, A2 \in \mathbb{C}^{n\times n}$ are general matrices.